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VIBRATIONAL BEHAVIOUR OF LATTICES OF PLATES: BASIC BEHAVIOUR AND HYPERSENSITIVITY PHENOMENA

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This paper deals with lattices of plates connected at a non-zero angle. The purpose is not only to identify the usual behaviour of lattices of plates (where pass-bands and stop-bands appear) but also to explain the rise of the amber-band. The notion of hypersensitivity existing for the case of two plates still exists for the case of a lattice of plates. When dealing with a lattice of identical systems, it is usual to observe the behaviour when some defects are included. This kind of study is proposed in this work, and some rules describing the rise of effects due to angular defects at a connection angle are proposed. A study focusing on a lattice of different coupled plates, is also proposed; a numerical approach shows results which could be suitable for an experimental tool detecting hypersensitive connections.

1. INTRODUCTION

The study of complex structures constructed from an assemblage of plates is of interest because such structures are often employed for mechanical, architectural or industrial applications. Some of these structures are made of several coupled identical plates and appear as a lattice. Even if the lattice is not infinite but constructed from a finite number of plates, its behaviour is typical.

In a previous paper [1] an analytical formulation for coupled plates describing the in-plate and flexural motions has been given. In this work this versatile tool is used to deal with structures which are lattices of plates.

By way of references some main results of the state of the art of lattice structures are cited and the contribution of this work to the subject is explained.

Periodic structures have largely been studied during the last forty years because of the great theoretical interest and the numerous industrial applications. Brillouin and Parodi [2] described in their study, from a comparison between lattices of different kinds (crystallography, electricity, mechanic), the basic laws of behaviour. In the same period, the first major study of the effects of imperfections (defects) had been carried out in solid state physics by Anderson [3]. Furthermore most of these results have been observed in mechanical structures of different basic systems: a one-degree-of-freedom system like a pendulum [4, 5] or a spring–mass system [6], axially vibrating roads [7], clamped beams used to represent the blades on a propeller [8], simply supported beams [9, 10] and plates in the same plane coupled by longitudinal stiffeners [10, 11].

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One can mention some usual results. A pass-band is a frequency band where the energy is spread all over the structure; on the other hand, a stop-band corresponds to an energy confinement inside the excited system. In between, Maidanik and Dickey [10, 11] proposed the term of amber-band which appears for finite structures.

As one can see, the papers dealing with lattices are considering more and more complicated basic systems in order to become more representative of industrial structures. This paper is following this track, plates with bending and in-plane motion, being taken as the basic system. The first goal of this work is to present the behaviour of lattices of plates, in order to verify if the vibration properties of periodic structures are still observed for such complicated basic systems and particularly to explain the formation of the amber-band. A second interest is to study the hypersensitivity phenomenon to clarify a main point directly issuing from reference [1]: does the hypersensitivity phenomenon observed when dealing with two coupled plates still exist with a lattice of plates? Lattices of plates with defects of the angles of connection, are also under study; the lattice considered is either constructed from identical plates or from different plates. For the latter case, a numerical approach shows results which could arise in experimental detection of hypersensitive connections.

2. MATHEMATICAL MODELLING OF A LATTICE OF PLATES

The mathematical modelling of a lattice of plates has been extensively described in reference [1]. Therefore a detailed description is not presented here; however, in the following, a basic description of the modelling is presented. The notation used is the same as in reference [1].

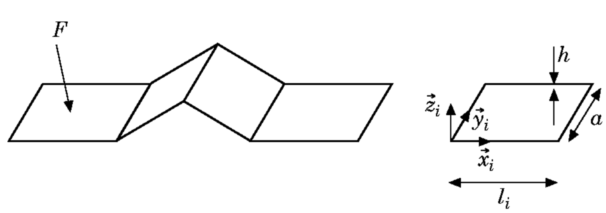
The structure under study is constructed from several thin isotropic plates of the same thickness and width but of different lengths, connected along the width at any angle. All the plates are simply supported on the lateral sides. The structure is excited by a pure tone force. A four-plates example is shown in Figure 1.

The Donnell operator for a shell of infinite radius, L_∞, is used to obtain the equations of motion for bending and in plane waves:

$$\frac{Eh}{1-\nu^2} L_\infty \begin{bmatrix} u \\ v \\ w \end{bmatrix} - \rho h \omega^2 \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \delta(x-x_0)\,\delta(y-y_0) \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}. \tag{1}$$

For the three displacements a semi-modal decomposition is used along the y direction and a wave formulation is adopted in the x direction. The general formulation, for example of the transverse displacement, is

$$w(x, y) = \sum_{n=1}^{\infty} W_n(x) \sin \frac{n\pi y}{a} \tag{2}$$



(a) (b)

Figure 1. (a) Example of a four identical plates coupled structure; (b) a particular plate of the structure.

and the modal amplitude is a wave function of x:

$$W_n(x) = A_n\,\mathrm{e}^{-k_{n1}x} + B_n\,\mathrm{e}^{k_{n2}x} + C_n\,\mathrm{e}^{-k_{n1}(l-x)} + D_n\,\mathrm{e}^{-k_{n2}(l-x)}. \tag{3}$$

For the coupled in-plane displacements along the x and y axes, the formulation is similar:

$$u(x, y) = \sum_{n=1}^{\infty} U_n(x) \sin\frac{n\pi y}{a} \qquad \text{and} \qquad v(x, y) = \sum_{n=0}^{\infty} V_n(x) \cos\frac{n\pi y}{a}. \tag{4}$$

The modal amplitudes are also wave functions of x, coupled together:

$$U_n(x) = F_n\,\mathrm{e}^{-k_{n3}x} + G_n\,\mathrm{e}^{-k_{n4}x} + H_n\,\mathrm{e}^{-k_{n3}(l-x)} + I_n\,\mathrm{e}^{-k_{n4}(l-x)} \qquad \text{and}$$
$$V_n(x) = -\lambda_{n3}F_n\,\mathrm{e}^{-k_{n3}x} - \lambda_{n4}G_n\,\mathrm{e}^{-k_{n4}x} + \lambda_{n3}H_n\,\mathrm{e}^{-k_{n3}(l-x)} + \lambda_{n4}I_n\,\mathrm{e}^{-k_{n4}(l-x)}. \tag{5}$$

For each fixed modal index n, there are eight unknowns per plate. Together with the boundary conditions which correspond to the continuity of motions, rotations, forces and moments between two perfectly coupled plates this gives rise to eight times the number of plates coupled equations. Solving this matrix equation for each mode allows one to calculate the total motion of the structure from a simple summation.

Two control parameters are used to qualify the vibrational behaviour of the structure: a global one and a local one. The mean transverse quadratic velocity of the ith plate is defined by

$$\langle V_i^2 \rangle = \frac{1}{2S_i}\int_{S_i} \omega^2 |w_i(M)|^2\,\mathrm{d}s. \tag{6}$$

where S_i is the surface of the ith plate. The mean transverse quadratic velocity of the whole structure is then:

$$\langle V^2 \rangle = \sum_{i=1}^{d} S_i \langle V_i^2 \rangle \Big/ \sum_{i=1}^{d} S_i. \tag{7}$$

The transfer mobility between two points A and B is the ratio between the normal velocity at B located on the ith plate, and the normal driving force located at A on the jth plate:

$$Y(A, B) = \mathrm{j}\omega w_i(B)/F_{zj}(A). \tag{8}$$

3. LATTICES OF IDENTICAL PLATES: VIBRATIONAL BEHAVIOUR

3.1. FINITE LATTICE OF IDENTICAL PLATES, BASIC RULES

This section is concerned with lattices of identical steel plates (density $\rho = 7{\cdot}85\ 10^3$ kgm^{-3}, Young's modulus $E = 2{\cdot}1 \times 10^{11}$ Nm^{-2}, Poisson's ratio $v = 0{\cdot}28$, damping coefficient $\eta = 0{\cdot}01$, length $l = 0{\cdot}5$ m, width $a = 0{\cdot}4$ m and thickness $h = 2 \times 10^{-3}$ m) all the connection angles are equal $\theta = 4^\circ$, and the plates are simply supported except on the connection edges.

The driving harmonic force is located on the first plate at $x_0 = 0{\cdot}3$ and $y_0 = 0{\cdot}17$. Two different lattices are under study: the number of plates is successively six and eighteen (see

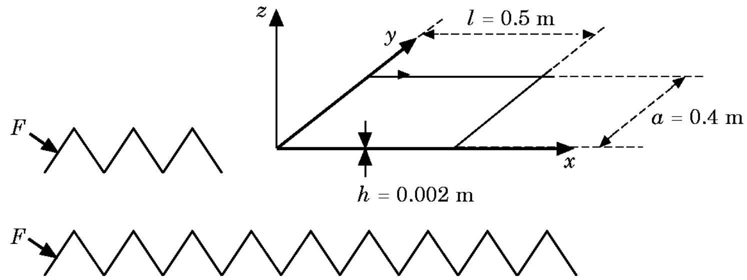


Figure 2. Lattices under study with six or eighteen identical plates connected with an angle of 4°, the driving force is located on the first plate at $x = 0{\cdot}3$, $y = 0{\cdot}17$. An example of one of the coupled plates with its size.

Figure 2). In Figures 3 and 4 are shown the quadratic velocities of each plate of the structure.

In Figure 4, the values of the quadratic velocity are very different (over a scale from -450 dB to -50 dB). Physically if the level of the quadratic velocity is -150 dB or -450 dB, the plate is practically motionless.

Typical pass-band and stop-band behaviour can be observed. One can locate an obvious pass-band for example at 142 Hz where the vibrational energy is equally spread all over the structure; indeed all plates are vibrating with almost the same kinetic energy. One can locate an obvious stop-band for example around 86 Hz where all the vibrational energy is confined to the first plate. In reference [7] Keane and Price showed that for an infinite lattice of one-degree-of-freedom systems, the amplitude of a wave traveling through the lattice decays exponentially at each connection if its frequency lies in a stop-band. The amplitude ratio of two adjacent systems is then a constant. The same behaviour occurs at 86 Hz; the quadratic velocity level, from one plate to the next, always decreases by 13 dB.

When dealing with a finite lattice of n coupled identical structures each with m degrees of freedom, the whole system presents m pass-bands, each pass-band having n peaks. The basic rule of vibration is of course respected: a system of M degrees of freedom (in this case $M = nm$) presents M eigenfrequencies. Here, one is considering plates with an infinite

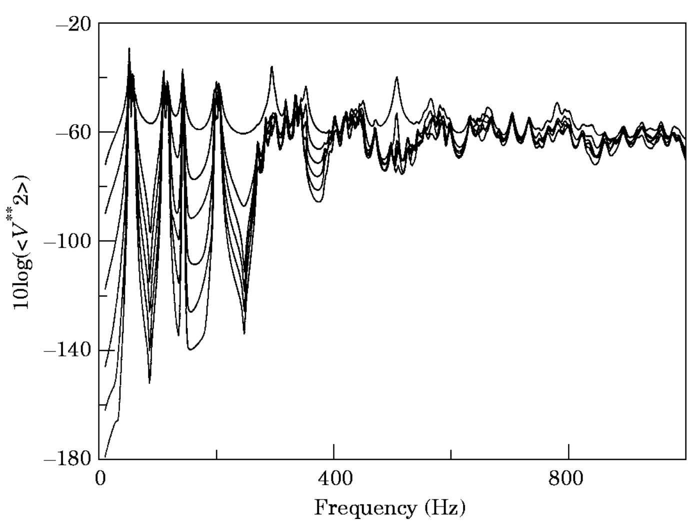


Figure 3. Quadratic transverse velocity of the six plates of the lattice, with an angle of connection of 4°.

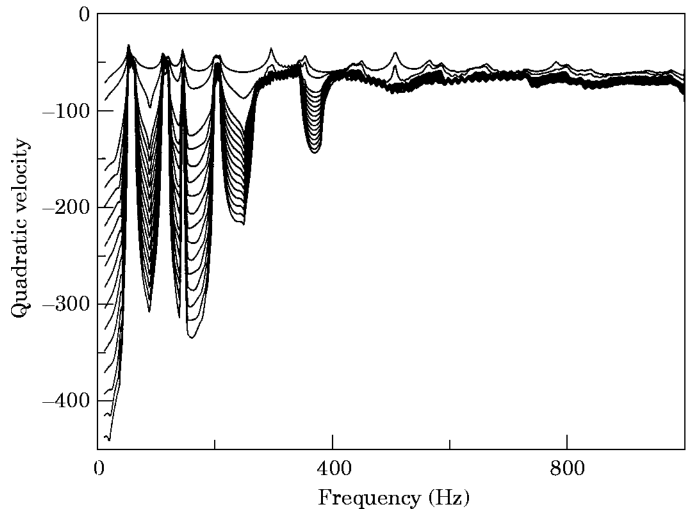


Figure 4. Quadratic transverse velocity of the eighteen plates of the lattice, with an angle of connection of 4°.

number of degrees of freedom, and one can suppose that the number of pass-bands is then infinite but in each pass-band the number of peaks is equal to the number of identical structures. In Figure 5 the quadratic velocity of the lattice constructed from eighteen plates is presented. One observes it around the first pass-band and decreases the damping ($\eta = 10^{-5}$) to avoid any modal overlap. One knows that there is no physical reality with such a small damping; it is just to be able to differentiate the peaks inside the pass-band. One can observe 18 peaks in the pass-band, as expected because of the 18 plates of the lattice. This behaviour was also observed for coupled beams in reference [9]. The lower frequency peak is equal to the first resonance frequency of a simply supported plate, the higher frequency peak tends to the resonance frequency of a plate clamped at junctions.

These two examples describe the overall behaviour of lattices of plates. The pass-bands and stop-bands are observed as in classical periodic structures but when the frequency

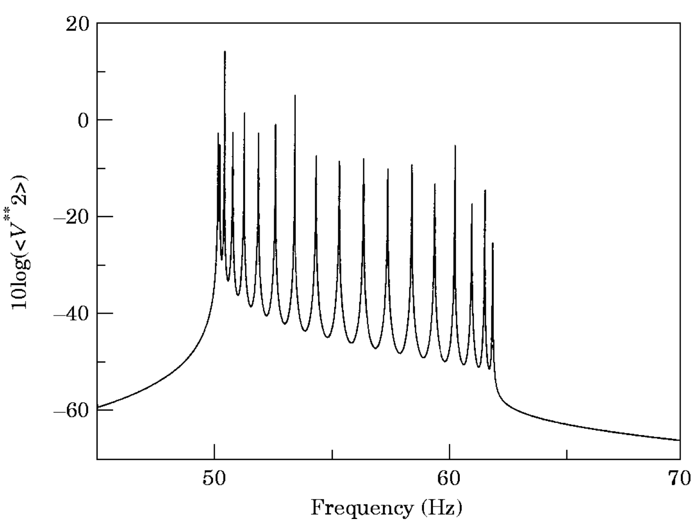


Figure 5. Total quadratic transverse velocity of the eighteen plates lattice, around the first pass-band, $\eta = 10^{-5}$.

increases there is also a third kind of frequency band—the amber-band—as will be discussed in the next section.

3.2. FORMATION OF THE AMBER-BAND

Maidanik and Dickey, to qualify their word "amber-band" said [11] that the behaviour of the structure in this frequency range lies between the one of a pass-band and the one of a stop-band, but did not explain its cause.

In Figure 4, above 600 Hz, one can then define this zone as an amber-band. The vibrational energy is neither equally distributed over the structure nor all confined to the first plate.

It is interesting to note that in the case of a lattice of beams vibrating in flexural motion, no amber-band is observed, but only stop-bands and pass-bands (see reference [9]). Thus the question is why the amber-band arises in the case of coupled plates and not in the case of coupled beams?

Through the semi-modal approach used in the analytical formulation, one can explain the rise of the amber-band. One notes that the motion of each plate is a superposition of modal motions in the y direction, each being associated to a wave in the x direction. Let the index of the modal expansion be fixed to a single value; the motion is then controlled by the wave in the x direction, and thus can be seen as that of a beam problem (or a one dimensional problem).

Figures 6(a)–(d) present the results for indexes fixed from 1–4, respectively; the classical lattice of beam behaviour, with only stop-bands and pass-bands, is observed. The lattice of plate behaviour, obtained by summation of the contributions of all the indexes is

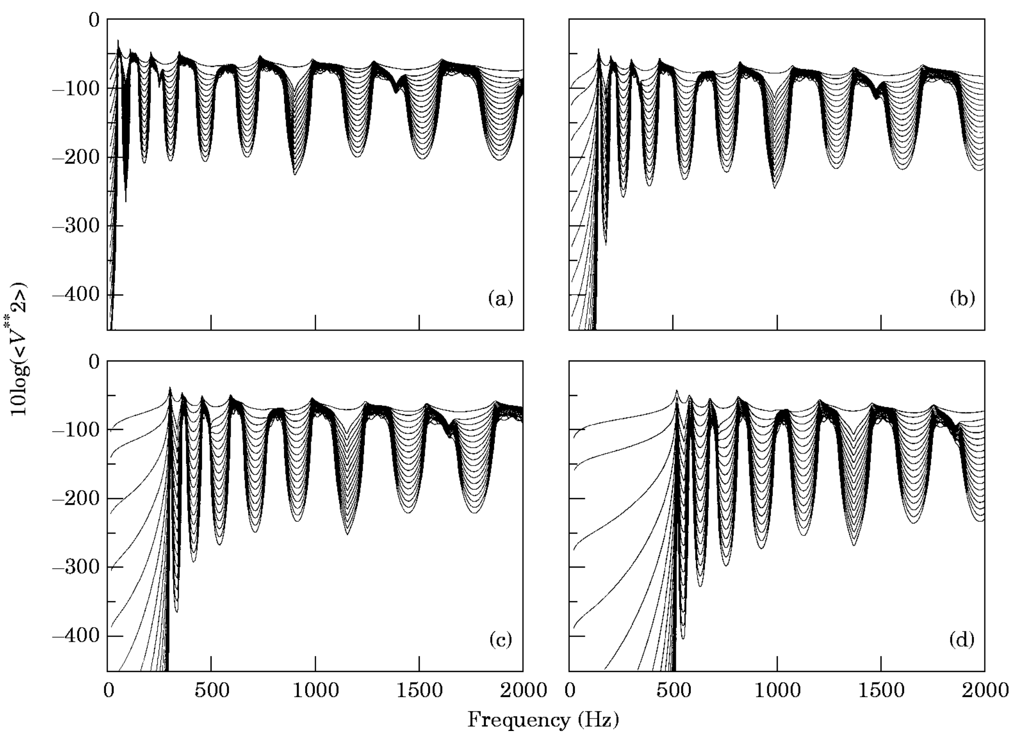


Figure 6. Quadratic modal transverse velocity of the eighteen plates of the lattice with an angle of connection of 90°. The modal index is respectively fixed at 1, (a); 2, (b); 3, (c); 4, (d).

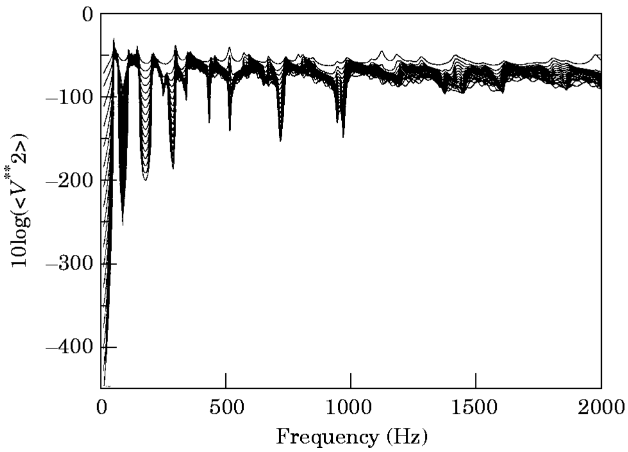


Figure 7. Quadratic transverse velocity of the eighteen plates of the lattice, with an angle of connection of 90°.

presented in Figure 7; the amber-band clearly appears above 1000 Hz as an overlap of the pass-bands associated to different indexes.

At low frequencies the pass-band overlap is impossible and the classical pass- and stop-bands are observed for plate lattices. When the frequency increases, the pass-band overlap becomes inevitable because of the increasing number of plate modes, and the lattice behaviour is of the amber-band type.

The previous results are obtained for plates coupled with an angle of 90°, allowing the decoupling of flexural and in plane motions. Because of transverse excitation one can say that only bending motion is present in the previous results.

For an angle of connection equal to 4°, flexural and in-plane motions are strongly coupled. The amber-band appears even when the index of the modal decomposition is fixed at a single value. This is of course due to the increase of the modal density resulting from the increasing number of flexural modes and in-plane modes. The behaviour of the lattice of plates coupled with an angle of connection equal to 4° has been already presented in Figure 4 and one can verify that the amber-band appears at a lower frequency than in the previous case of 90° connection angle (600 Hz instead of 1000 Hz).

To summarize, one can take the simple case of multi-supported coupled beams in flexural motion. The pth pass-band is located between the pth simply supported beam resonance frequency (the sketch of the motion is presented in Figure 8(b)) and the pth clamped resonance frequency (the sketch of the motion is presented in Figure 8(c)). Thus the pth pass-band Δ_p is located in the frequency band defined by

$$\Delta_p = [\omega_p^{ss}, \omega_p^{c}]. \tag{9}$$

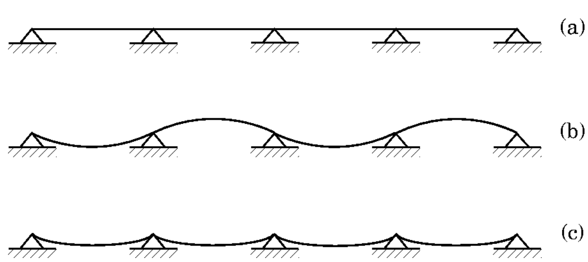


Figure 8. First pass-band of a multi-supported beam. (a) Sketch of the multi-supported beam; (b) sketch of the lower frequency motion in the first pass-band (simply supported resonance frequency); (c) sketch of the higher frequency motion in the first pass-band (clamped resonance frequency).

The well known result concerning the resonance frequencies of beams shows that $\omega_p^c < \omega_{p+1}^{ss}$, and in consequence pass-band overlap is impossible as numerically evidenced in reference [9] and in the previous Figures 6(a)–(d).

For beam-like structures the amber-band will arise only when coupling between different types of vibration is present; typically of flexural and longitudinal motions.

As a general simple rule one can say that the amber-band will appear when the width Δ of individual pass-bands becomes greater than the modal separation δ of the structure: $\Delta > \delta$. By introducing the modal density n as the inverse of the modal separation, the condition of appearance of the amber-band can be expressed as $\Delta n > 1$. Even if it has been observed for a finite lattice, as here, this rule is also true for an infinite lattice.

3.3. HYPERSENSITIVITY OF BEHAVIOUR OF LATTICES

This section is concerned with four lattices of eighteen plates. The plates are the same as previously. For the first structure, all connection angles are 4°, for the second 6°, for the third 25° and for the last one 90°. Two similar lattices will be compared, the first with the second, and two distinctly different lattices, the third with the fourth.

Figures 9(a) and 9(b) present the quadratic velocity spectra of each plate when the connection angle is 4° and 6°, respectively. For both it is easy to identify pass-bands and

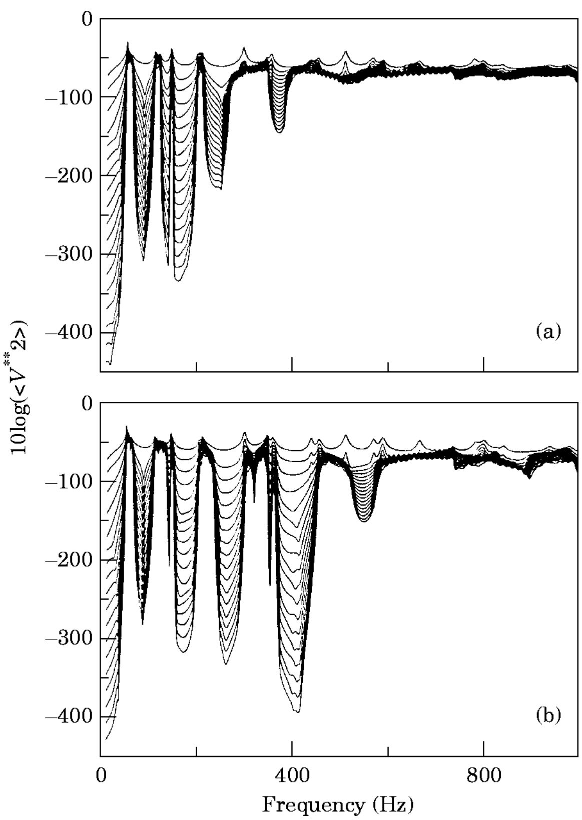


Figure 9. Quadratic transverse velocity of the eighteen plates of the lattice, with an angle of connection of (a) 4° and (b) 6°.

stop-bands; it is important to note the differences in the locations. For example, 400 Hz for the first structure is not a particular frequency and for the second it corresponds to a stop-band. This sensitivity to the modification of the connection angle between plates is explained by the strong variation of the coupling between bending and in-plane motion effects. This notion of hypersensitivity has been expressed and discussed in reference [1].

On the other hand, when the angle is large, the coupling between the two kinds of motion is no longer sensitive, even to a large variation of the angle. For example when the angle of connection is 25° or 90°, the two lattices which are physically very different have close vibrational behaviour (see Figures 10(a) and 10(b)). Physically, at these large angles, the effects of in-plane motion are negligible.

Through these four examples, one notes that the first pass-band is approximately in the same place. A deeper study of this frequency area is proposed. A very low damping coefficient is used again ($\eta = 10^{-5}$) to allow each peak to be perceptible.

For the first couple of structures where the angles are 4° and 6° respectively (see Figure 11), the pass-band is wider for the larger angle. The coupling with in-plane motion is stronger with the connection angle of 6° and increases the stiffness of the structure.

For the next two structures where the angles are 25° and 90° respectively (see Figure 12) the widths and the positions of the eigenfrequencies are the same; the stiffness of the structure does not vary significantly as a result of the decoupling of bending and in-plane motions.

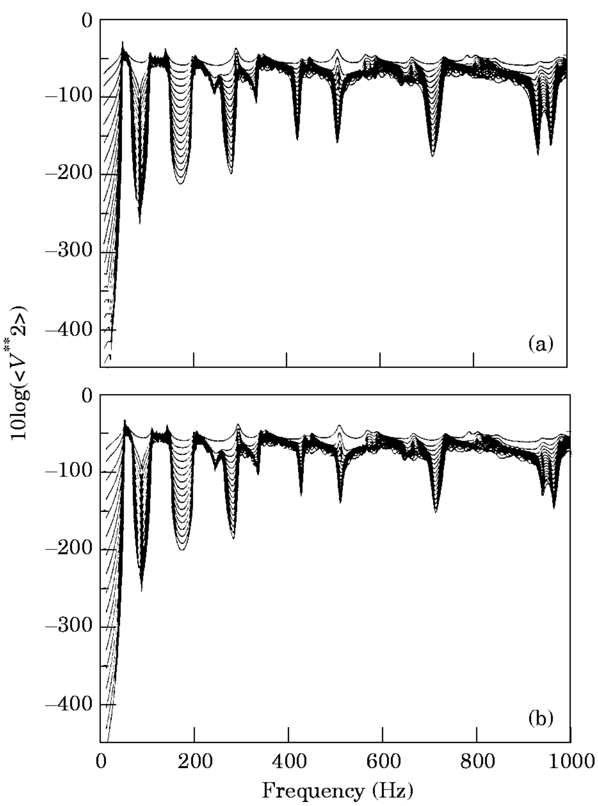


Figure 10. Quadratic transverse velocity of the eighteen plates of the lattice, with an angle of connection of (a) 25° and (b) 90°.

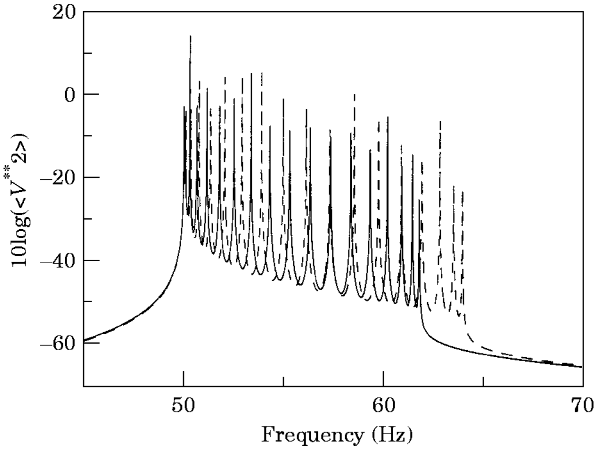


Figure 11. Quadratic transverse velocity of the whole structure, on the first pass-band. Damping coefficient is $\eta = 10^{-5}$. Solid line, angle of connection of 4°, dashed line, angle of connection of 6°.

The aim of this section was to answer the question if the properties of hypersensitivity to the connection angle as described in reference [1] when dealing with two plates still exist when dealing with a lattice of plates. Obviously hypersensitivity still exists. The coupling between in-plane motion and bending is strongly variable when the angle of connection is small; a weak modification of it can bring strong variations in the vibrational behaviour: this property is not dependent on the number of plates. In addition, the lattice has no real effect of amplifying or reducing the phenomenon.

3.3. LATTICES WITH DEFECTS

According to reference [4], to exhibit the "Anderson localization", which is a result of a defect on a perfect lattice, the ratio (disorder/coupling) between systems has to be important. In the present lattices, the coupling between plates is very strong and it is impossible to exhibit Anderson localization. Nevertheless the effects of defects can be observed in a different way as will be described in this section.

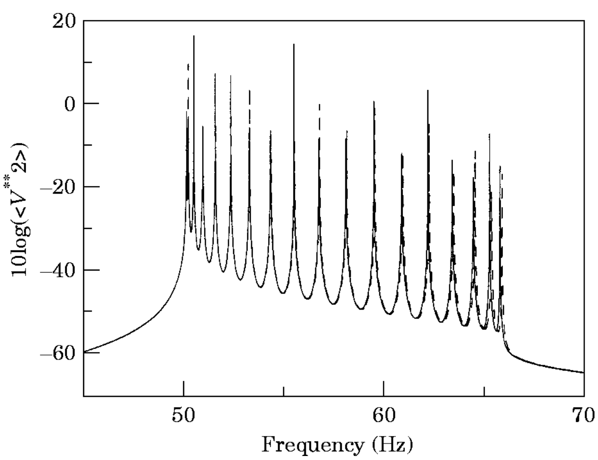


Figure 12. Quadratic transverse velocity of the whole structure, on the first pass-band. Damping coefficient is $\eta = 10^{-5}$. Solid line, angle of connection of 25°, dashed line, angle of connection of 90°.

The control parameter is no longer the quadratic velocity, which is too global, but the transfer mobility between the driving force and the middle of each plate; however, to make trends appear this quantity will be averaged over frequency.

Consider two structures. The first one is a lattice without defect and the second one has a defect. For each plate of the altered structure the relative error of the transfer mobility magnitude at a fixed frequency is defined by

$$Er_i(\omega) = \sqrt{([|\bar{Y}_{ij}(\omega)| - |Y_{ij}(\omega)|]/|\bar{Y}_{ij}(\omega)|)^2}, \tag{9}$$

where i is the receiving plate index and j the excited plate index, $\bar{Y}_{ij}(\omega)$ is the transfer mobility modulus for the reference structure without defect and $Y_{ij}(\omega)$ the transfer mobility modulus for the altered one. To make trends appear, observation at a particular frequency is not convenient, and hence an average over frequency to smooth the phenomenon can be used. The mean value over the angular frequency band Δ is defined by

$$\langle Er_i \rangle_\Delta = \frac{1}{\Delta} \int_{\omega_c - \Delta/2}^{\omega_c + \Delta/2} Er_i(\omega)\, \mathrm{d}\omega, \tag{10}$$

where ω_c is the centre of the frequency band.

Only the effects of a defect in the angle of connection will be observed in this section. The location, the amplitude and the number of defects can change. The structures under study are constructed from eighteen plates, whose characteristics are the same as previously and the driving force is on the first plate.

3.4.1. *The effect of one defect on the connection angle*

Consider the structure constructed from eighteen coupled plates with an angle of 4°, in which one angular defect is imposed on the junction between plate 1 and plate 2. This connection angle is successively 4·1°, 4·5° and 5° and all other angles remain fixed at 4°. The relative error in the transfer mobility magnitude versus the plate index is plotted in Figure 13. The bigger the amplitude of the defect, the greater are the effects. Through this figure it is very easy to find the location of the defect; it corresponds to the peak on the

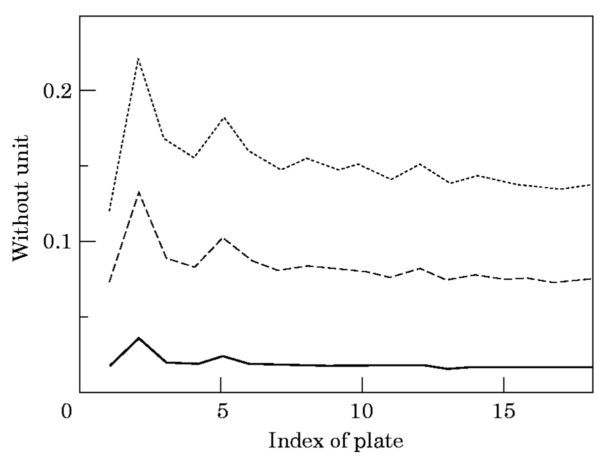


Figure 13. Effects of an angular defect set on the first connection of the lattice of plates coupled with an angle of 4°. The angular defect is respectively 0·1°, solid line; 0·5°, dashed line; and 1°, dotted line.

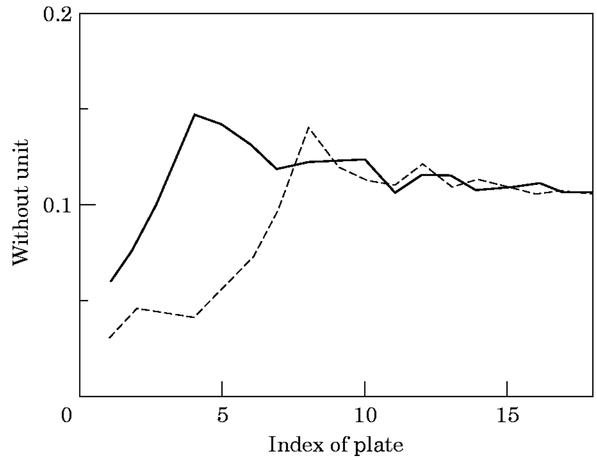


Figure 14. Effects of an angular defect of 1° applied to a lattice of eighteen plates coupled with an angle of 4°. The angular defect is respectively set on the third connection (solid line) and on the seventh connection (dashed line).

value of $\langle Er_i \rangle_\Delta$ after this peak the value remains constant. These particularities are also illustrated in Figure 14: two lattices of eighteen plates connected at 4° are considered where an angle of 5° is put firstly on the third connection (between plates 3 and 4) and secondly on the seventh connection (between plates 7 and 8).

For the previous examples, the defect was always set on a connection where the angle is hypersensitive as defined in reference [1]. It is then interesting to study a structure which is a quasi-lattice: namely, there are eighteen identical plates, all connected with an angle of 4° except the junction between the plates 5 and 6 where the angle is 40°.

When the defect of 1° is applied on the fifth junction, the effects are negligible because at this junction the angle of connection is not hypersensitive. The contrary result occurs when the same defect is applied on a hypersensitive connection angle. Results for $\langle Er_i \rangle_\Delta$, for these two examples, are plotted in Figure 15.

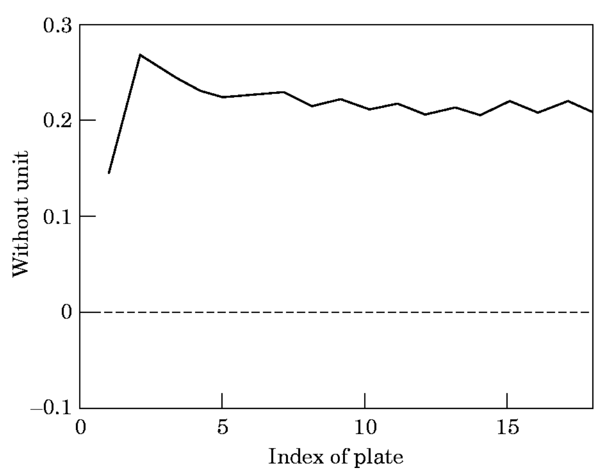


Figure 15. Effects of an angular defect of 1° applied to a quasi-lattice of eighteen plates coupled with an angle of 4°, but 40° between plates 5 and 6. The defect is respectively located on the first connection (solid line) and on the fifth connection (dashed line).

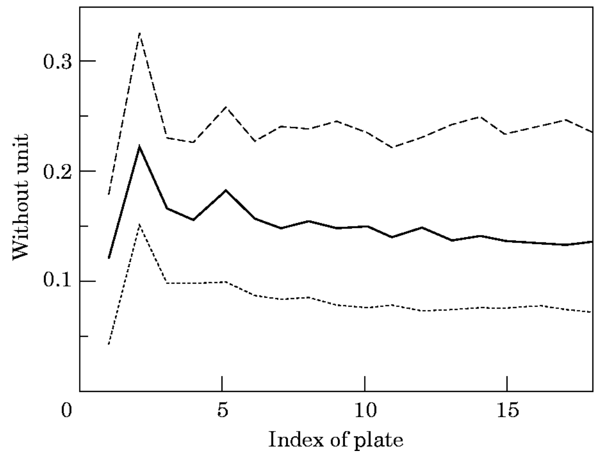


Figure 16. Effects of an angular defect of 1° set on the first connection of the lattice of eighteen plates coupled with an angle of 4°. The damping coefficients is respectively $\eta = 10^{-1}$ (dotted line), $\eta = 10^{-2}$ (solid line) and $\eta = 10^{-3}$ (dashed line).

As a matter of conclusion, one can say that, even if several connection angles are hypersensitive, the hypersensitivity of behaviour is observed only if defects are directly applied to hypersensitive angles of connection.

The damping of the structure is simulated as structural damping through a complex Young modulus, $E^* = E(1 + j\eta)$. One illustrates the effects of the same defect on a lattice of eighteen plates connected at 4° with an η that is successively 10^{-1}, 10^{-2} and 10^{-3} (keep in mind that 10^{-2} is the "usual" value of damping). The defect corresponds to an angle of 5° between plates 1 and 2. Results are presented in Figure 16. When the damping increases, the values of $\langle Er_i \rangle_\Delta$ are lower; this can be easily explained as follows. A defect on a connection angle leads to a frequential shift of the peaks, and $\langle Er_i \rangle_\Delta$ is the average of the differences at each frequency. Then if the levels of the peaks of the transfer mobility magnitude are lower because the damping increases, the coefficient $\langle Er_i \rangle_\Delta$ decreases. Figure 17 is a sketch to illustrate this.

From these few examples, one can identify some properties: the defect gives rise to effects only when it is located on a hypersensitive angle; the coefficient $\langle Er_i \rangle_\Delta$ allows one to detect a defect on an hypersensitive angle; the effects are proportional to the value of the defect; the effects propagate all over the structure after the location of the defect.

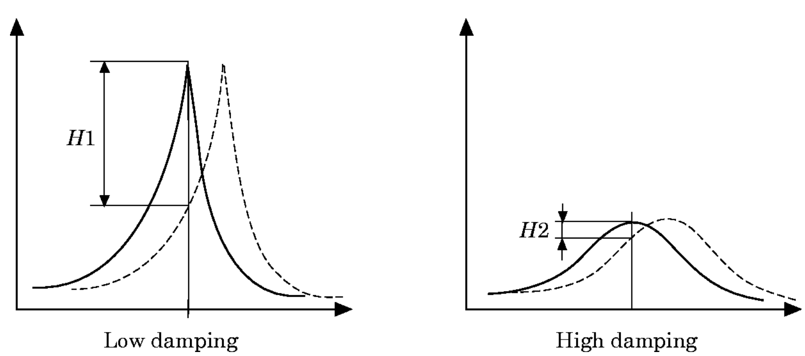


Figure 17. Principle of the decreasing of effects of one defect as damping increases; effects are more important as damping decreases ($H1 > H2$).

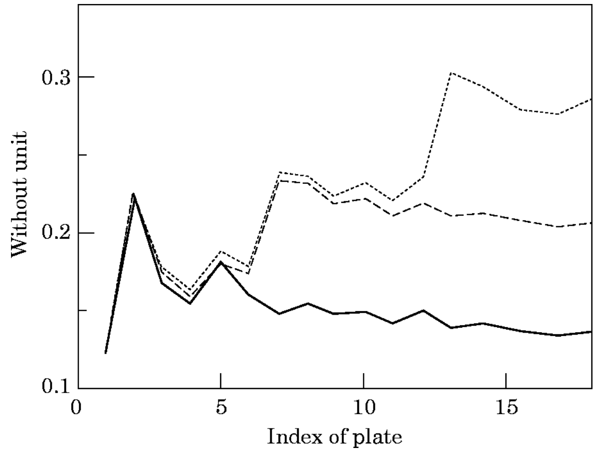


Figure 18. Effects of some angular defects of 1° applied to a lattice of eighteen plates coupled with an angle of 4°. One angular defect, on the first connection (solid line); two, on the first and on the sixth connections (dashed line); three, on the first, on the sixth and on the twelfth connection (dotted line).

3.4.2. *The effect of several defects*

The basic lattice used previously (eighteen plates connected with an angle of 4°) is used once again. Three different cases of defects are compared. The first structure presents one defect (angle of 5° between plates 1 and 2), the second two (angle of 5° between plates 1 and 2, and 6 and 7) and the third three (angle of 5° between plates 1 and 2, 6 and 7 and 12 and 13). Values of the coefficient $\langle Er_i \rangle_\Delta$ for these examples are plotted in Figure 18.

The properties previously observed with one single defect remain. In addition, one can observe a kind of additivity of the influence of defects. The effects of each defect—constant and propagated all over the structure after the defective connection—are added to the effects of the previous defects; that is why one can observe steps on the curves. With curves like the one in Figure 18, one can easily find the location of the defects.

4. NON-PERIODIC LATTICE OF PLATES

Up to now the lattices of the plates considered were constructed from identical plates. Consider now a non-periodic lattice of plates. The main reason for such a study is that several industrial structures can be modelled as a non-periodic lattice, or in other words as coupled plates. So hypersensitivity basic rules for industrial structures must be preferably studied on non-periodic lattice to avoid special behaviour due to periodicity.

Hence, in this section, a lattice of nine different plates connected at different angles is considered. The plates have the same width (0·4 m) and the same thickness (2 mm), the structure presented in Figure 19 could be, for example, the hood of a machine. From what has been learnt from the previous results one can guess that there are three connections which are hypersensitive; they are marked in Figure 19.

4.1. EXISTENCE OF THE HYPERSENSITIVITY

The goal of this part is to verify if one's hypothesis about the hypersensitivity is right. Hence, one imposes successively on each connection angle an angular defect of 1°. The comparison with the perfect structure through the indicator $\langle Er_i \rangle_\Delta$, as defined previously, is presented in Figure 20. It is obvious that the effects of an angular defect of 1° are

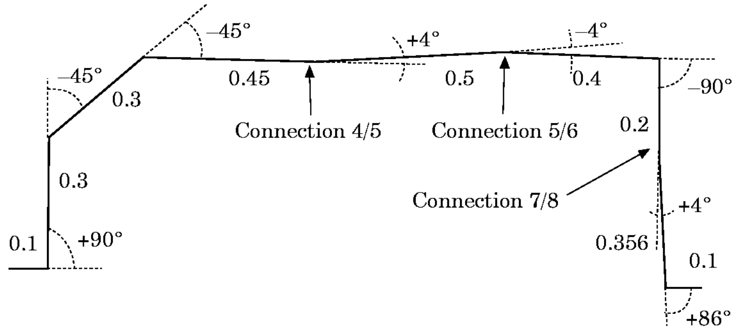


Figure 19. Lattice of nine non-identical plates, as an industrial structure such as the hood of a machine. The three connections marked are presumed to be hypersensitive.

perceptible when located on a hypersensitive connection and not when located on another connection. It is possible to see the difference between the two kinds of connection angles.

In this way one can conclude that the hypersensitivity phenomenon still exists when dealing with a non-periodic lattice of plates.

The phenomenon of hypersensitivity exists whenever plates are coupled, whatever the number and the configuration of the coupled plates.

4.2. SIMULATION OF AN EXPERIMENTAL APPROACH

A weak angular variation on a hypersensitive connection can lead to an important variation of the behaviour. As has been said, this is due to a modification of the coupling between in-plane and normal displacements. Can this modification be also produced by any kind of structural modification? And particularly, when considering a mass defect added to a connection, does it produce the same effect as an angular defect? To get an answer to this question is relatively important because it will be possible to show that the hypersensitivity phenomenon is really due to the structure and not to the kind of defect.

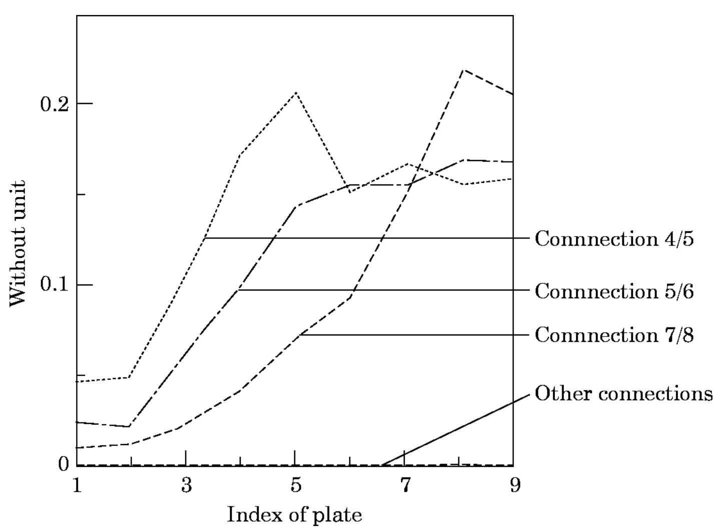


Figure 20. Effects of an angular defect successively put on each connection between plates; identification of the hypersensitive connections.

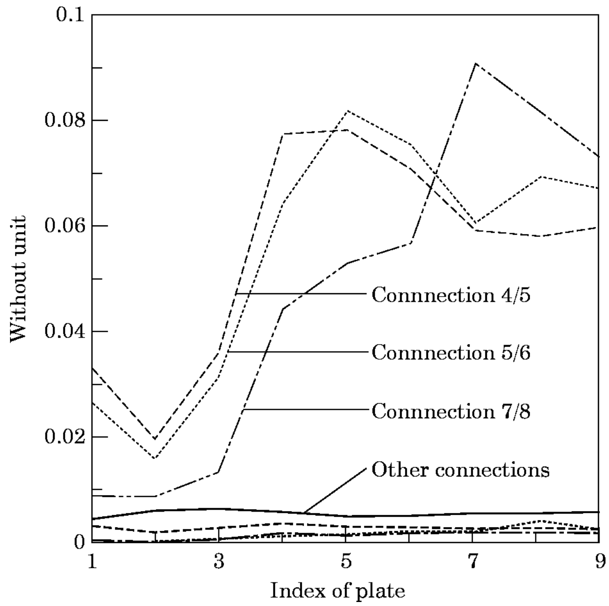


Figure 21. Effects of a mass defect successively put on each connection between plates; identification of the hypersensitive connections.

Another point has to be considered. When dealing with a real structure it is difficult to add an angular defect on one of the connections of the structure without damage to the structure. On the contrary one can manage to add a temporary mass defect on one of the connections which will not affect the long term integrity of the structure.

Consider the structure previously presented and add successively a light mass defect on each connection angle. The added mass is really light (0·0628 kg) in comparison to the mass of the whole structure (16·68 kg) and also in comparison to the mass of each plate (varying from 0·628 kg to 3·14 kg). In the mathematical modelling, the added mass is simulated by a narrow plate with increased density along the connection. One can also use the same way of thinking as was done for the angular defect: the modification applied to the structure is still weak. Comparisons between the different slightly damaged structures and the perfect one is observed through the parameter $\langle Er_i \rangle_\Delta$ already defined and used previously.

The results are presented in Figure 21. They are comparable to the previous results. When a mass defect is added on a hypersensitive connection the levels of the effects are really higher than when it is added on another connection.

This proves that the hypersensitivity phenomenon is intrinsic to the structure, because two different kinds of defects (angular defect or added mass defect) give the same conclusions.

One also, in this way, simulates results which could be obtained from an experimental approach, and the main interest in this is that, by the use of a mass defect as a non-destructive approach, one can identify hypersensitive connections of a lattice of plates.

5. CONCLUSION

The analytical formulation previously developed in reference [1] for the vibrational behaviour of coupled plates has been applied to cases of lattices of plates.

The usual behaviour of lattices has now been observed for the case of coupled plates. It has been verified that a pass-band corresponds to a frequency range where vibrational energy is equally spread over the structure, and a stop-band corresponds to a spatial confinement of the energy into the excited plate. In addition to these two typical bands there is a third one, called the amber-band, where the energy is neither located inside the excited plate nor spread over the structure. It is shown that the rise of an amber-band corresponds to a pass-band overlap, which appears when the size of the pass-band is greater than the modal separation (inverse of the modal density).

The notion of hypersensitivity observed in the case of two coupled plates is also observed to be applicable when dealing with lattices of plates. Two close lattices can present strong differences, of the vibrational behaviour (when the angle is small); otherwise two quite different lattices can present practically no differences in vibrational behaviour (when the angle of connection is large).

The Anderson localization phenomenon appears when a defect is included in a perfect lattice and when the ratio (defect/coupling) is high [4]. As a strong coupling between plates has been modelled here, it is not possible to exhibit it. But the effects of angular defects on the connection have been checked. It appears that a defect gives rise to effects only if located on a hypersensitive angle, and that the effects of one defect are proportional to its amount, which spreads to the rest of the structure located after it, and is cumulative with other defect effects.

Finally, considering non-periodic lattices has allowed a study of structures close to industrial reality. The simulations of angular and mass defects show two main results. The hypersensitivity phenomenon exists whenever plates are coupled (identical or different) and is basically linked to the structure and not to the kind of defect.

Some numerical simulation results have been presented, which indicate the possibility of an experimental non-destructive approach to identification of the hypersensitive connections of a lattice of plates.

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